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NUMERICAL ESTIMATION OF STRESS INTENSITY
FACTORS BY A FINITE DIFFERENCE EULERIAN
METHOD (THE HELP CODE)

Prepared by

Rockwell International Corporation
Missiles Systems Division
4300 East 5 Avenue
Columbus, OH 43216

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BALLISTIC RESEARCH LABORATORY
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I. INTRODUCTION

This report details the initial steps of a modest effort to explore failure criteria suitable for incorporation in two- and three-dimensional computer codes for analysis of high velocity impact and penetration phenomena. Such codes have shown over the years their ability to successfully treat impact problems in various velocity regimes and have been most notably successful in the area of hypervelocity impact (loosely definable as a regime where pressures generated in impacting bodies exceed the material ultimate stress by several orders of magnitude). While existing codes can indicate high stress areas which are likely locations for occurrence of fracture, none can handle the fragmentation of an initially intact penetrator or target into individual fragments and then track the deformation and motion of those fragments. Such a capability is still far off and will require refinements in material characterization, failure initiation and propagation.

It is now generally well established that failure of materials subjected to intense impulsive loads is a time-dependent phenomenon. But existing codes, with few exceptions, deal only with simplistic criteria which assume instantaneous failure of material in a computational cell or element once the critical stress or strain is exceeded. Models offering greater realism are available and are reviewed briefly in the paper by Jonas and Zukas¹.

¹Jonas, G. H. & Zukas, J. A., "Mechanics of Penetration: Analysis and Experiment", Int. J. Engng. Sci., Vol. 16, pp 879-903, 1978.

We can distinguish among three distinct approaches to fracture characterizations; continuum models based on cumulative damage concepts, models concerned with microscopic aspects of ductile and brittle fracture, and crack propagation models. In this report, we focus our attention on the crack propagation models.

The means to limit the damage to a structure subjected to ballistic impact or maximize it in the case of offensive interactions is of great practical importance in the design of many structures. Both require a knowledge of the response of the target to impact. One of the keys to understanding energy dissipation in a structure as it experiences damage lies in a knowledge of dynamic fracture phenomena which can relate the kinetic energy of the incoming projectile to subsequent modes of failure, distortions, and heat that may characterize the damage. The total available energy may partially be dissipated in the creation of new surfaces either by nucleating new cracks or by extending the existing ones. In the latter case the available energy can be related to the crack driving force and therefore to the fracture stress intensity. The need to account for such phenomena is clear since many penetrators of practical importance have either super- or sub-caliber grooves machined into the rod to permit fitting of a sabot. For high obliquity impacts, these grooves act as stress raisers and frequently serve as fracture initiation sites. The subsequent reduction of the penetrator mass can severely impair its performance, especially against multi-plate target. Additionally, imperfections in materials due to poor quality control in production processes may impair

ballistic performance. This is easily noticeable in Lambert's report² in which the data are compared for two cases involving oblique impact of 65 gram steel penetrators with length-to-diameter ratio of 10 against rolled homogeneous armor plates. A change in the manufacturing processes to reduce the inclusion rate in the steel rods dramatically reduced scatter in the data resulting in an improved estimate of ballistic limit with fewer rounds. In other areas, the presence of cracks and inhomogeneities in confined explosives subjected to projectile impact may be important sources of hot spots leading to violent reactions^{3,4}.

The existence of such inhomogeneities is currently ignored in most computer simulations of impact. The incorporation of the Stanford Research Institute NAG-FRAG model⁵⁻⁸ in two- and three-dimensional codes

²Lambert, J. P., "The Terminal Ballistics of Certain 65 Gram Long Rod Penetrators Impacting Steel Armor Plate", BRL Technical Report ARBRL-TR-02072, May 1978. (AD#A057757)

³Engineering Design Handbook, Principles of Explosive Behavior, AMCP 706-180, April 1972.

⁴Chawla, M. S. and Frey, R. B., "A Numerical Study of Projectile Impact on Explosives", BRL Memorandum Report 2741, April, 1977. (AD#A040433)

⁵Barbee, T. W. et al, Dynamic Fracture Criteria for Ductile and Brittle Metals. J. of Materials, JMLSA, Vol. 7, No. 3, September 1972.

⁶Seaman, L. and Shockey, D. A., "Models for Ductile and Brittle Fracture for Two-Dimensional Propagation Calculations", Army Materials and Mechanics Research Center Final Report AMMRC CTR 75-2, February 1975.

⁷Shockey, D. A. et al, "A Computer Model for Fragmentation of Armor Under Ballistic Impact", BRL Contract Report 222, April 1975. (AD#B004672L)

⁸Hageman, L. J. and Herrmann, R. G., "Incorporation of the NAG-FRAG Model for Ductile and Brittle Fracture in HELP, A 2D Multi-Material Eulerian Program", ARBRL-CR-00380, September 1978. (AD#A062335)

will, in the future, partially offset this deficiency. But NAG-FRAG is a nontrivial and expensive model and requires considerable material characterization before it can be applied to realistic situations. Hence, examination of simpler models is desirable.

It had been the initial objective of this work to look at fracture criteria involving the J-integral and the crack opening displacement (COD) type with a view towards incorporating such a capability in HELP and EPIC Codes (the primary codes used at the BRL for kinetic energy penetrator impact studies) and comparing the results with existing instantaneous criteria for oblique impact of notched penetrators and normal impact of confined, nonhomogeneous explosives. With the departure of the first author and a change in the workload this objective can no longer be met. This report therefore documents the first step taken in this process, a study of the feasibility of performing a HELP code calculation with included cracks and voids. A second purpose of this report is to obtain a feeling towards the accuracies of the HELP code results compared to the previously published results obtained by utilizing the highly dependable HEMP code, for a standard example.

II. DISCUSSION

A crack in a plate becomes unstable when the stress state in the immediate vicinity of the crack tip becomes critical. For a brittle material, the linear elasticity theory is generally adequate in describing the macroscopic effects such as displacements and the stresses at

distances large compared to the crack radius. According to this theory, the stresses in the neighborhood of a crack tip show a $\frac{1}{\sqrt{r}}$ type behavior^{9,10,11} where r is the radial distance measured from the crack tip. Consequently, the stress components obtain singular values at the crack tip indicating a breakdown of the elastic models in the vicinity of the crack tip. In reality, most materials have a small plastic zone in the neighborhood of the crack tip, so that the stresses do not actually become infinite at the crack tip. Nevertheless, significant stress enhancement does take place due to the presence of a crack which may lead to a catastrophic failure under certain conditions. To overcome the difficulties in the analysis that an infinite stress causes, it is customary to normalize the stresses by multiplying them by a function of \sqrt{r} . The limiting value of the product function is called stress intensity factor. For dynamic problems, a dynamic stress intensity factor can be similarly defined.

While analytic solutions for crack problems are available for many configurations¹², they are limited to certain idealized geometries and static load conditions. Hence it is important to develop a methodology which is simple in concept while being capable of handling complicated

⁹G. R. Irwin, Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate, J. Appl. Mech, Trans. ASME 24, 361, 1957.

¹⁰G. R. Irwin, "Fracture" Encyclopedia of Physics, Springer-Verlag 6, 558 (1958)

¹¹I. N. Sneddon, Proc Royal Soc London, 187, 229 (1946)

¹²G. C. Sih and J. F. Loeber, Wave Propagation in an Elastic Solid With a Line Discontinuity or Finite Crack, J. Acoust. Soc. Amer. 44, 90, 1968.

geometries under dynamic impact conditions. Two computer programs, HELP¹³ and EPIC¹⁴ have been tested for the purpose of solving a simple but nontrivial problem of dynamic fracture mechanics. The HELP code is a two-dimensional, finite difference Eulerian code which can deal with elastic-plastic solids as well as pure hydro problems. This code is especially good in handling problems involving extreme distortions. In the HELP code, free surfaces and interfaces are handled in a pseudo-Lagrangian fashion. The pressure, stresses and velocities are all cell-centered quantities. Tillotson equations are employed to describe the state of the material. In addition, the Von-Mises yield condition describes the onset of plasticity and a maximum volumetric strain defines the material failure. In the present problem, the plasticity and the material failure are not considered. In this paper we will report the results of the test using the HELP Code. The EPIC Code Solution will be attempted in the near future.

¹³L. J. Hageman, D. E. Wilkins, R. T. Sedgwick and J. L. Waddell, "HELP: A Multimaterial Eulerian Program for Compressible Fluid and Elastic-Plastic Flows in Two Space Dimensions and Time", Systems, Science and Software Report No. SSS-R-75-2654, 1975.

¹⁴G. R. Johnson, "EPIC-3, A Computer Program for Elastic-Plastic Calculations in 3 Dimensions", BRL Contract Report 343, July 1977. (AD#A043281)

III. PROBLEM SET UP

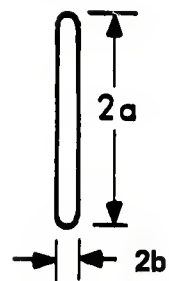
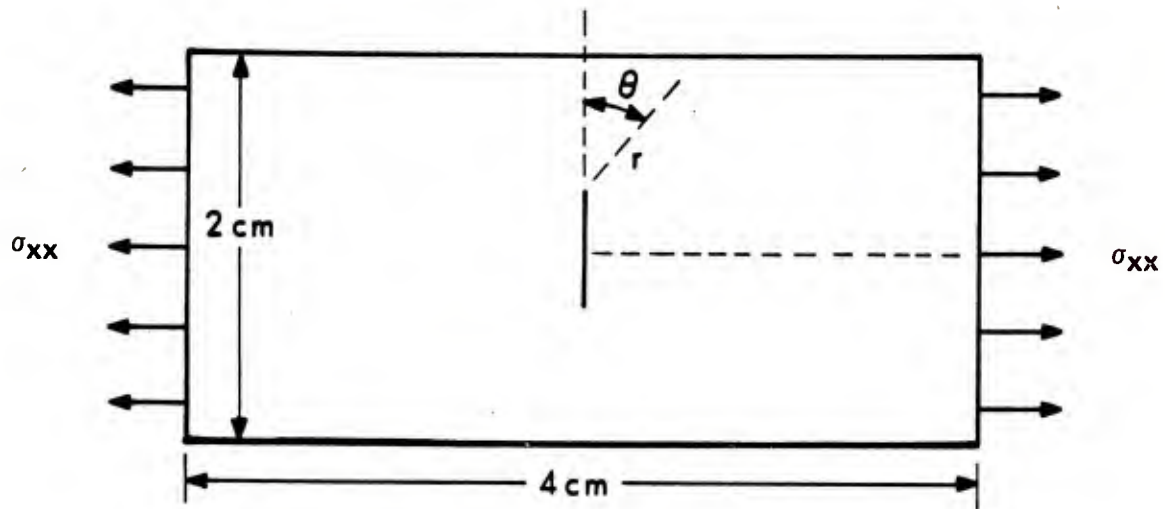
The study reported here involved a rectangular plate in plane strain geometry loaded in uniaxial tension. Similar problem involving a different material has been addressed by Chen¹⁵ and Chen and Wilkins¹⁶, using the Lagrangian HEMP code. The plate is shown in Figure 1 and the loading is shown in Figure 2. The elastic constants for the plate material are listed in Table I.

TABLE I. ELASTIC CONSTANTS OF MATERIAL

Density, ρ	= 7.80 g/cc
Bulk Modulus, κ	= 1.27×10^{12} dyne/cm ²
Shear Modulus, G	= 1.93×10^{12} dyne/cm ²
Logitudinal sound speed, $V_L = \sqrt{\frac{\kappa + \frac{4}{3} G}{\rho}}$	= 0.702 cm/ μ s
Transverse sound speed, $V_T = \sqrt{\frac{G}{\rho}}$	= 0.497 cm/ μ s
Bulk sound speed, $C_0 = \sqrt{\frac{\kappa}{\rho}}$	= 0.403 cm/ μ s
Rayleigh sound speed, $V_R = 0.495 V_L$	= 0.347 cm/ μ s

¹⁵Y. M. Chen, Numerical Computation of Dynamic Stress Intensity Factors by a Lagrangian Finite-Difference Method (The HEMP Code), Eng. Fract. Mech. Vol. 7, pp. 653-660, 1975.

¹⁶Y. M. Chen and M. L. Wilkins, Elastodynamic Crack Problems, Noordhoff International Publishing, Leyden, 1977.



$$a = .260$$

$$b = .033$$

Figure 1. The Plate Geometry

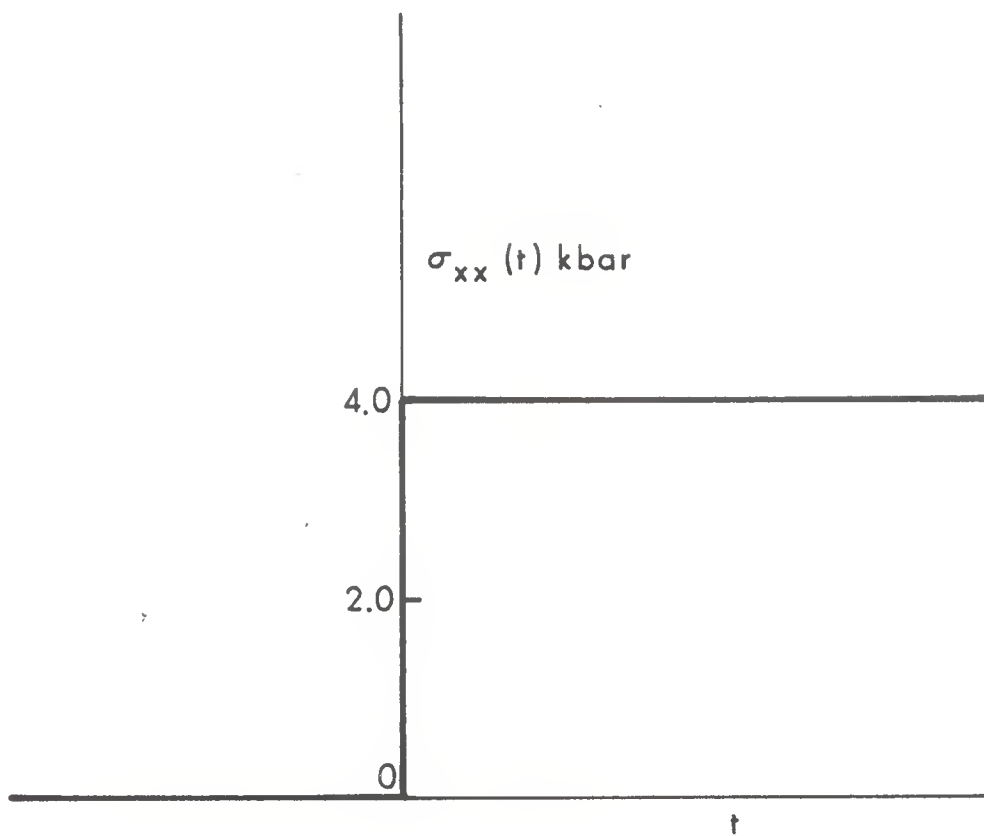


Figure 2. The Load Applied to the Rectangular Plate

The elastic constants listed in Table I perhaps do not represent any real material. The high value of the Shear Modulus corresponds to a Poisson's ratio of zero. Substantial simplification of the material stress state is thus obtained by not letting any waves arise from transverse plate contractions or crack opening. For the configuration shown in Figure 1, it is sufficient to solve the quarter problem because of the mirror symmetries along the $x=0$ and the $y=0$ planes.

Figure 3 is a schematic of the first quadrant of the problem. The entire grid consists of 40 cells in the x -direction and 47 cells in the y -direction, as shown in Figure 3. The first three columns of the cells define the half width of the crack, which is nearly invisible in the figure because of its small size. The cells are ultrafine near the crack tip. Additionally these cells are square and have constant area so that numerical errors near the crack tip are minimal. In the rest of the sample, where the accuracy was of less concern, the cells are not square and cell sizes increase progressively as one goes away from the crack tip along the x - or y -axis. The HELP code was modified to accommodate the present example for which the loading was tensile instead of compressive, which in turn necessitated defining the initial conditions in terms of pressure rather than velocity as is customarily done in the HELP code for solution of impact problems. Here the material was assumed to be perfectly elastic. Extreme caution was exercised to define the circular slot edge. The quarter circular arc of the slot was defined by means of 40 straight line segments.

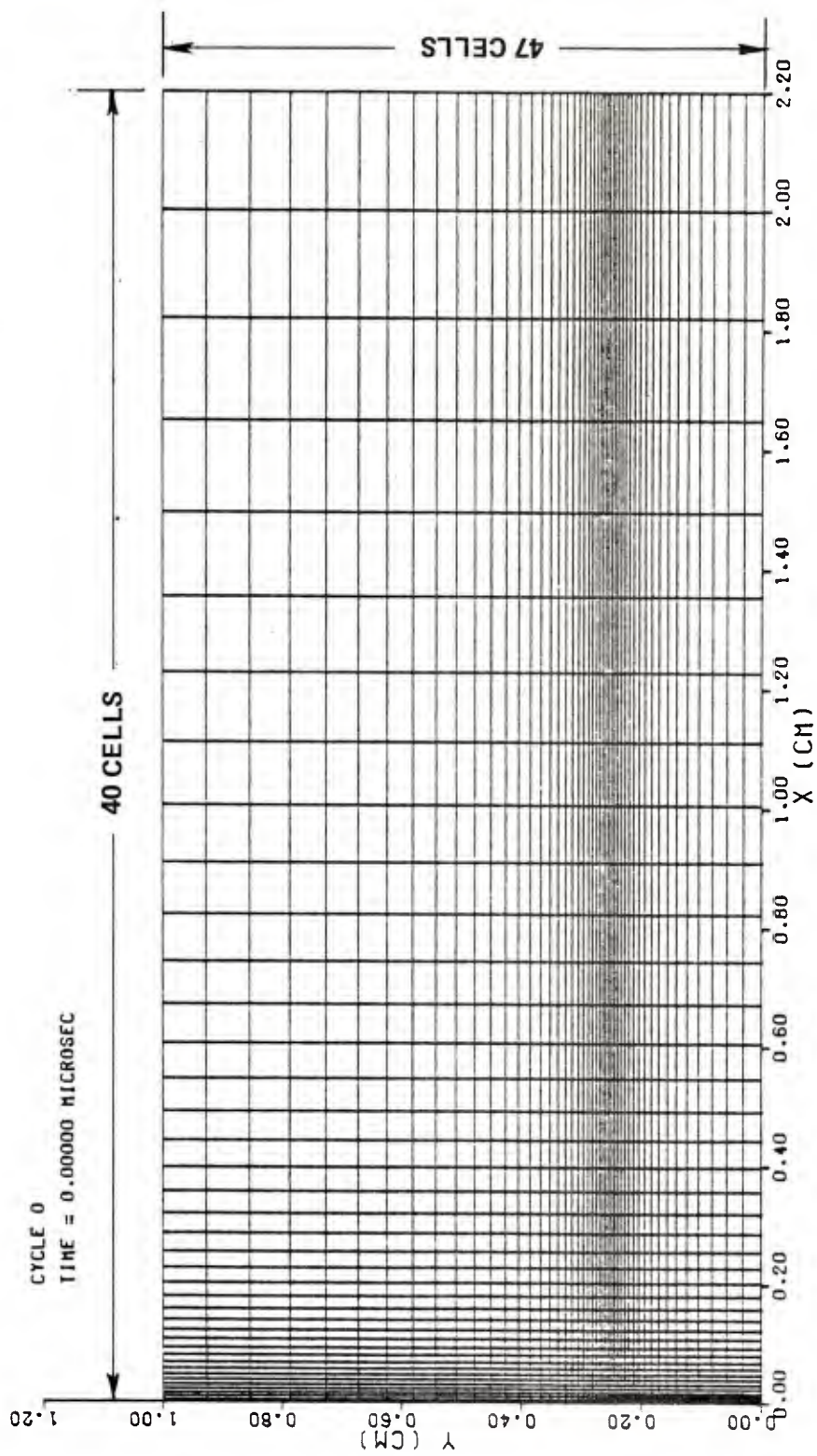


Figure 3. The Grid Employed to Study the Quarter Plate

IV. RESULTS

The stress intensity factor for the elastic material in the vicinity of the crack tip is defined by

$$k_1(t) = \lim_{r \rightarrow 0} k_1^*(t) \quad (1)$$

where

$$k_1^*(t) = \sqrt{\frac{2\pi r}{f(\theta)}} \sigma_{xx}(r, \theta, t) \quad (2)$$

and

$$f(\theta) = \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) \cos \frac{\theta}{2} \quad (3)$$

The last two equations are based on the results of the static field equations describing the stress state of a perfectly elastic medium in the vicinity of a crack⁹⁻¹¹. Existence of the corresponding solution for the dynamic case has been assumed. Equation (1) indicates a limiting process which yields the crack-tip stress intensity factor utilizing the near field results. The rigorous process indicated in Equation (1), however, cannot be carried out in practice since the finite-difference code calculations yield incorrect stress values (σ_{xx}) near the crack tip. Instead an extrapolation method can be used for estimating the stress intensity factors. Assumption is made that $\sigma_{xx}(r, \theta, t)$ is correctly defined a few crack radii away from the crack tip. Extrapolation path is chosen along the line, $\theta = 0$, in accordance with Chen's suggestions. The function $k_1^*(t)$ is defined for points along $\theta = 0$ for different values of r . It was found that $k_1^*(t)$ define a straight line of the type

$$k_1^*(t) = k_1(t) + \beta(t)r \quad (4)$$

A least subroutine was employed to successively exclude points from the fit until the change in the sum of residuals was less than 5%. This usually amounted to ignoring the first seven or eight points near the crack tip. The straight line thus obtained was extrapolated to $r = 0$, yielding an estimate of $k_1(t)$ from Equation (4). Figure 4 shows the extrapolation technique for a given time. This technique was employed

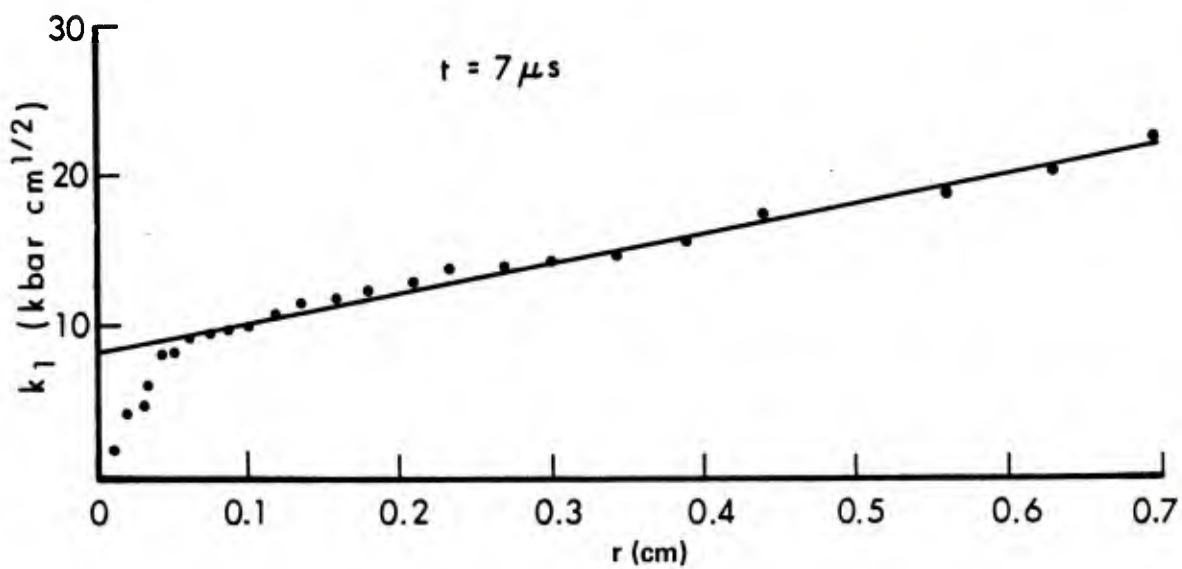


Figure 4. A Typical Linear Extrapolation Curve for k_1

to generate all the data points for $0 \leq t \leq 18 \mu s$ which are plotted in Figure 5. This figure shows several oscillations. It is interesting to try to attempt to correlate these oscillations with the arrival of waves reflected from various free surfaces. The times for arrival of various waves are tabulated in Table II and are also marked on Figure 5. All the times are measured from the instant when the load is applied, and were calculated simply by dividing the appropriate space dimension by the corresponding wave velocity.

TABLE II. Times of Arrival of Various Scattered Waves
at the Crack Tip as Marked in Figure 5

OA = $2.85 \mu s$	Arrival of first L wave at the crack tip
OB = $4.35 \mu s$	Arrival of Rayleigh wave reflected from the other crack tip
OC = $4.96 \mu s$	L reflection from the free surface
OD = $5.83 \mu s$	T reflection from the free surface
OE = $8.55 \mu s$	Arrival of L wave reflected from the other end
OF = $10.05 \mu s$	Arrival of T wave reflected from the other end
OG = $10.66 \mu s$	Reflection of the second L wave
OH = $11.53 \mu s$	Reflection of the second T wave

Clearly, an attempt to correlate the oscillations in k_1 values with the arrival times of various waves has been met with mixed success. This is hardly surprising because the finite difference codes characteristically tend to smear out sharp wave fronts. The wave arrival times are therefore difficult to pick out in a hydrocode solution.

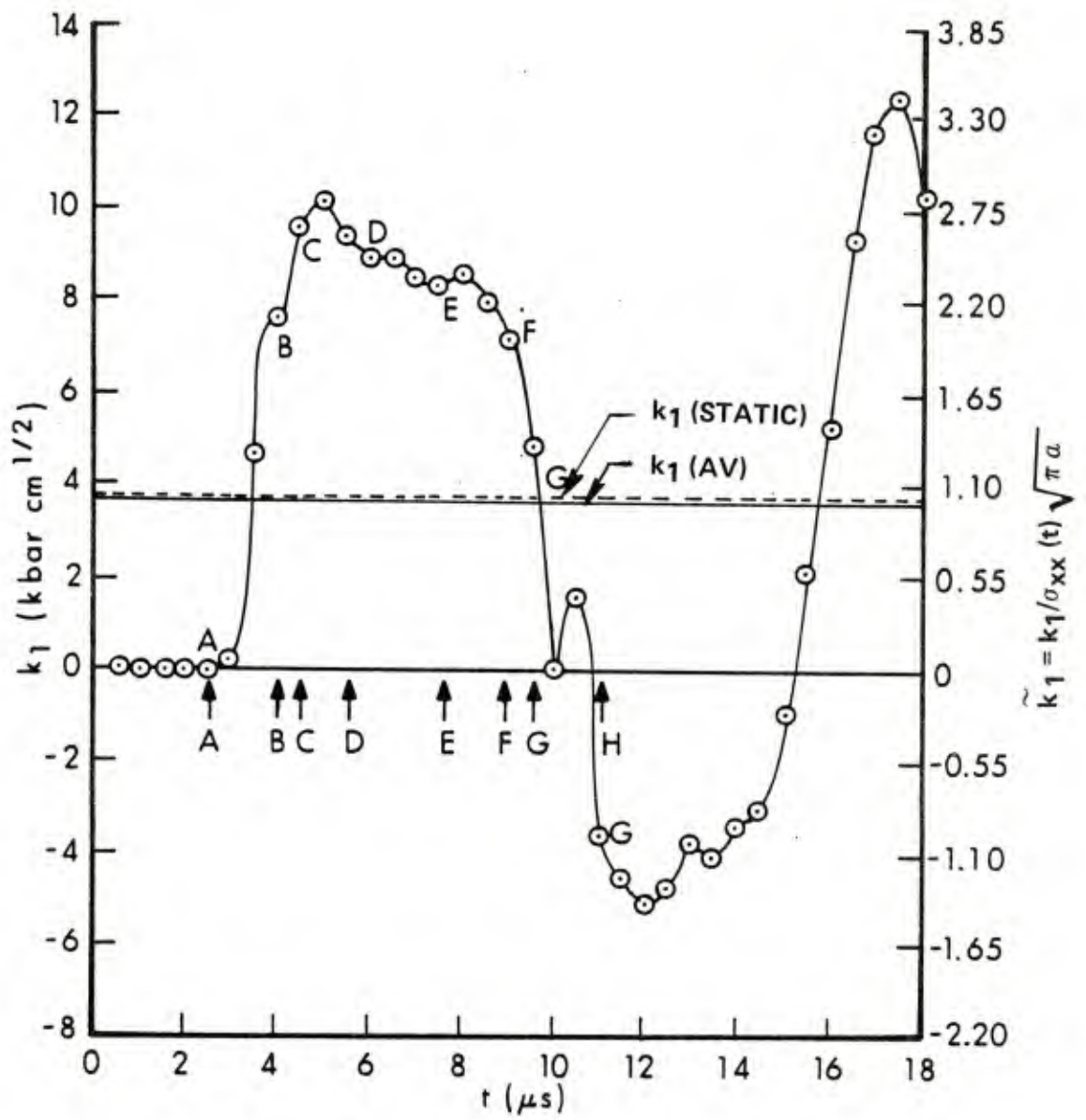


Figure 5. Stress Intensity Factor versus Time

The plots of σ_{xx} versus t for two observation points near the crack tip are shown in Figure 6. It is clear that as the distance of the point of observation from the crack tip decreases, the absolute value of σ_{xx} increases. However, for points very near the crack tip, the σ_{xx} values will be inaccurate. We feel that our case, the σ_{xx} values are accurate for $r \geq 0.05$ cm. A number of oscillations are also apparent in the σ_{xx} versus t curve. These oscillations, like the ones in Figure 5, result from cancellation and reinforcement of the scattered waves from various free surfaces. It should be pointed out that if there were no cracks, the σ_{xx} curve will oscillate about the mean value, $P(t) = 4$ kbar. The plot in Figure 7 shows the computer results for the same geometry without cracks, while Figure 8 shows the stress transverse to the crack, σ_{yy} , for a given time for the entire grid. The time-average of $\sigma_{xx}(t)$ versus t plots shown in Figure 6 is a roughly 5.2 kbar. The average is customarily obtained by considering the first global minima and the second global maxima which are believed to be not significantly altered by the dynamic oscillations. The first global maxima is thus ignored from the averaging process. The average σ_{xx} value corresponds to a time of $15.75 \mu s$. The k_1 value corresponding to this time is $3.75 \text{ kbar} \sqrt{\text{cm}}$. This compares remarkably well with the time average 3.73 obtained from the plot in Figure 5 or the theoretical value¹⁷ 3.75 or Chen's value¹⁵ 3.72. As in Chen's case, the present result also agrees with Baker's result¹⁸ for a load of $2P(t)$. Also since Baker employed an infinite plate, the agreement will vanish on arrival of the first reflected wave at the crack tip.

¹⁷ A. S. Kobayashi, R. D. Cheerpy and W. C. Kinsel, A Numerical Procedure for Estimating the Stress Intensity Factor of a Crack in a Finite Plate, Trans. ASME, Series D, 86, 681, 1964.

¹⁸ B. R. Baker, Dynamic Stresses Created by a Moving Crack, J. Appl. Mech. 29, 449, 1962.

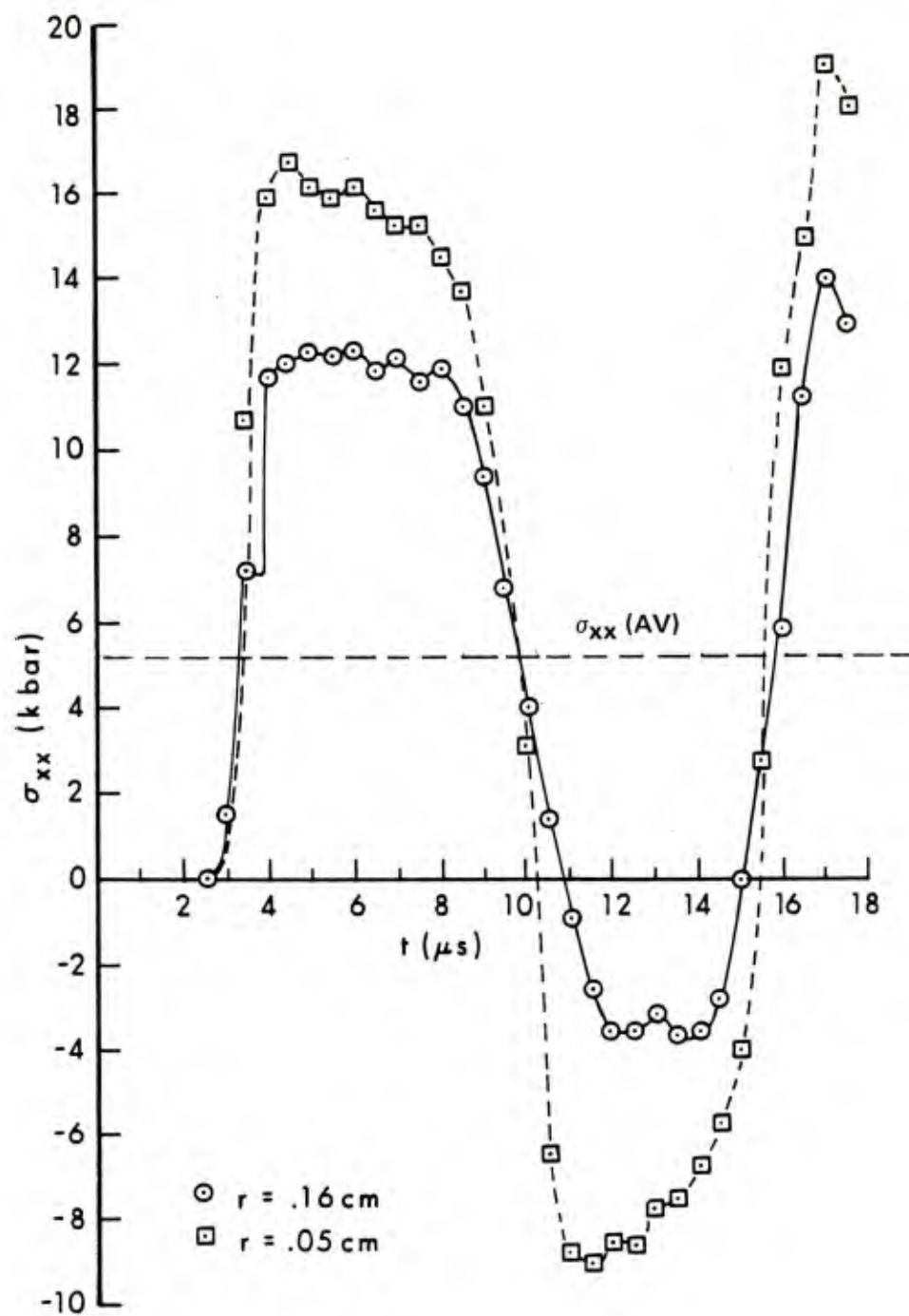


Figure 6. Transverse Stresses versus Time at Points Slightly Away from the Crack Tip

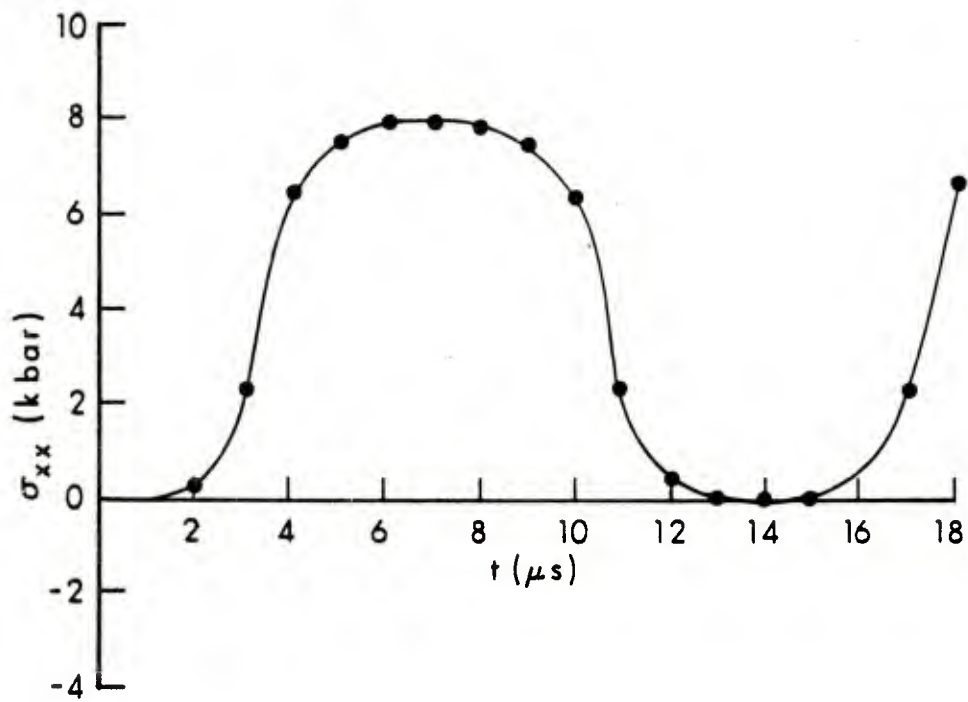


Figure 7. Transverse Stress versus Time at the Center of the Uncracked Plate

TRANSVERSE STRESS (kbar)
 CYCLE 830
 TIME = 4.000 MICROSEC

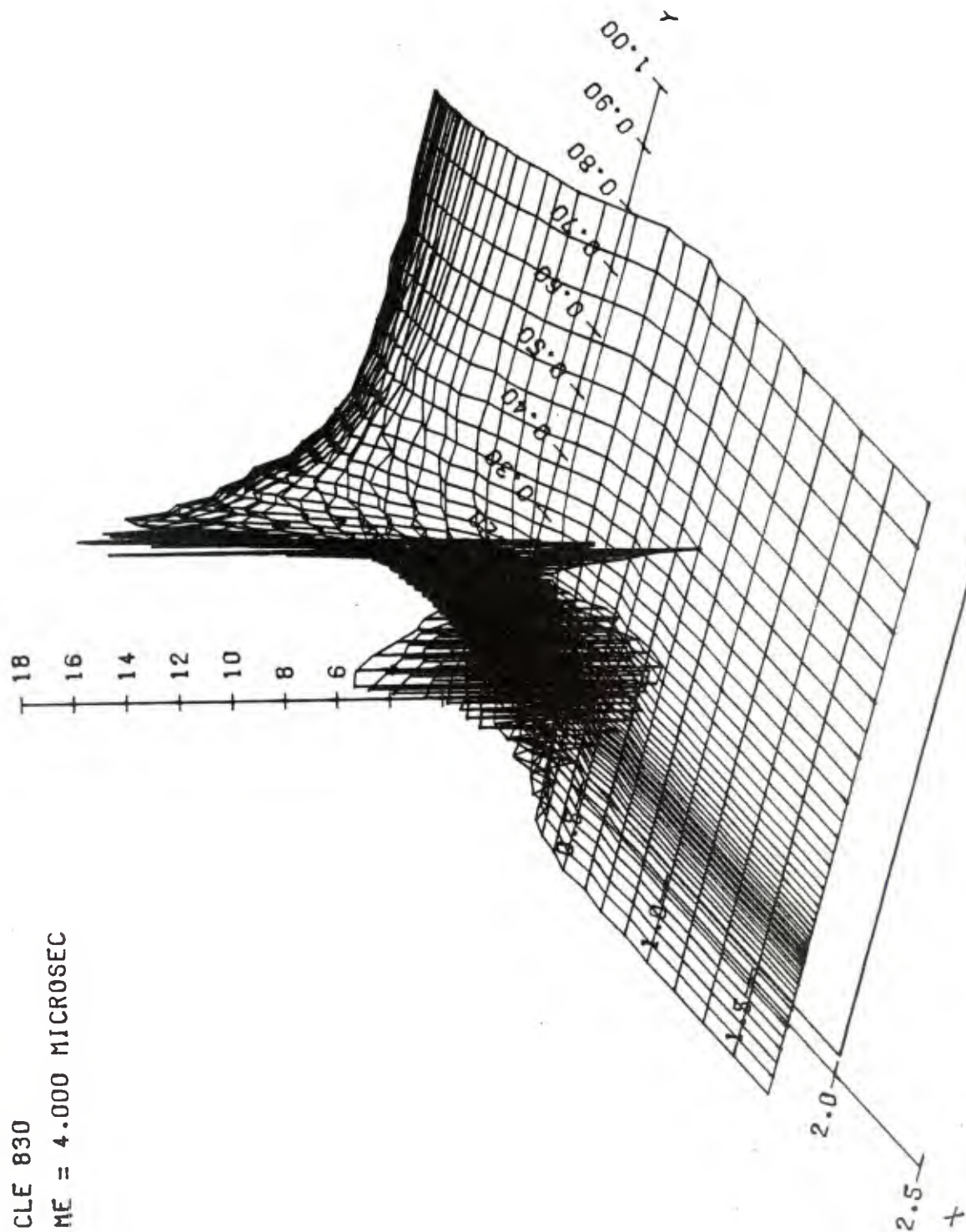


Figure 8. Transverse Stresses in the Quarter Plate at time $t = 4.0 \mu s$

Figure 9 shows the crack openings at various times. The crack extension and new surface creation are not modeled in the present study. If desired, the crack tip movement can roughly be followed by placing a few master/slave particles in the y axis, along which the extension is expected to take place provided some critical conditions are met. The crack in our example does not seem to open up in the x axis as freely as in Chen's case. This is due to the fact that the material employed in this study has higher rigidity modules than in Chen's case. Figure 10 depicts the change in the crack width as a function of time. Crack width increases when the wave is tensile, but it is never seen to decrease below its original value at $t = 0$. The transverse stress seldom becomes strongly compressive near the middle of the crack as can be seen from Figure 6.

V. CONCLUSIONS

Based on the above results, one can conclude that as long as the loading is in the elastic regime, an extrapolation method can be employed to calculate the dynamic as well as the static values of the stress intensity factors. These values can then be employed to study the stability of cracks when used in conjunction with some suitable crack propagation theory. The procedure outlined above can be extended to complicated geometries undergoing extensive deformation as a result of impact. Either HELP or HEMP code can be employed to study the crack stability. This example also shows that stresses as well as the details of material motion are qualitatively so similar in the two codes - one Eulerian, other Lagrangian, that it can hardly be dismissed as fortuitous.

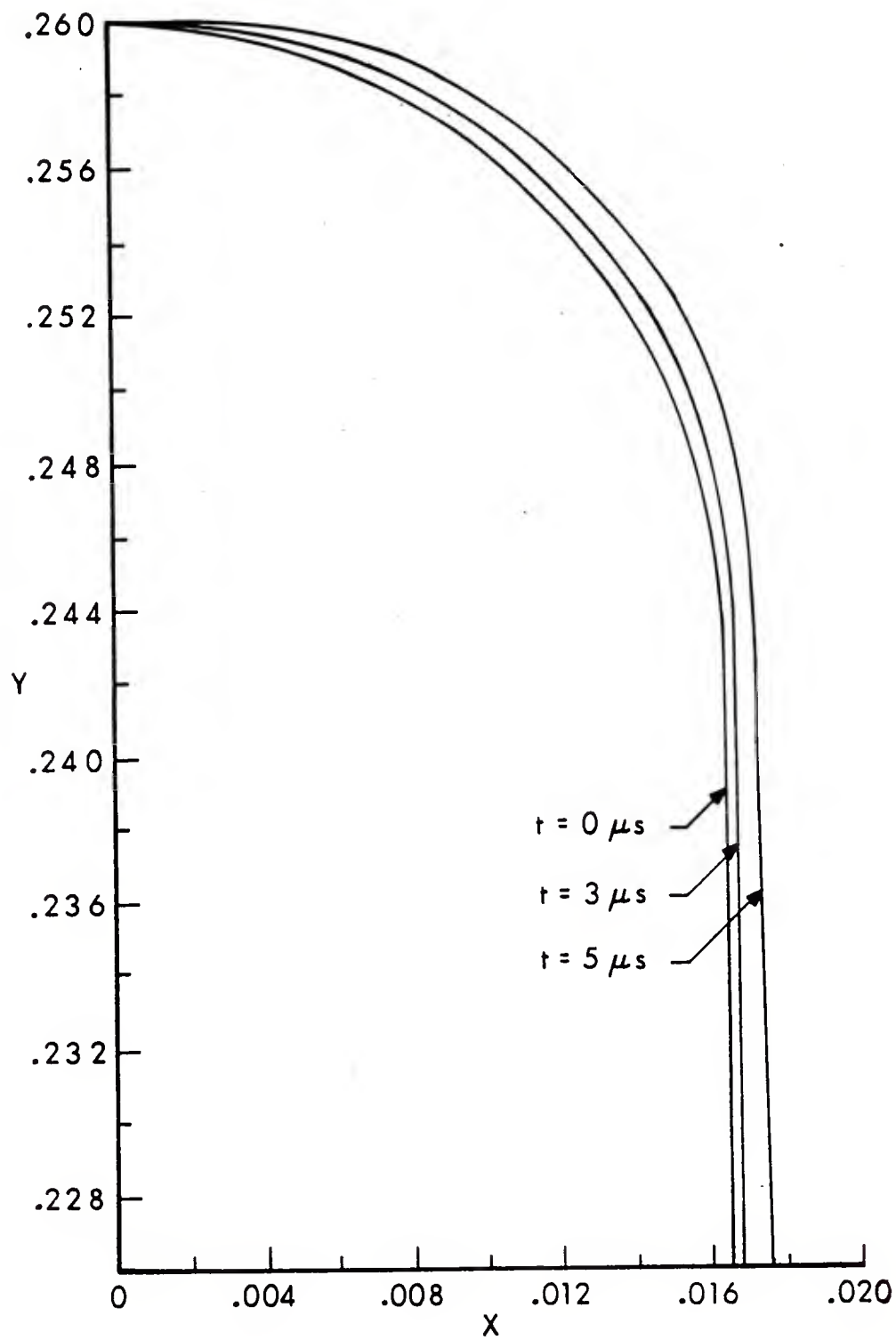


Figure 9. Crack Opening at Three Different Times

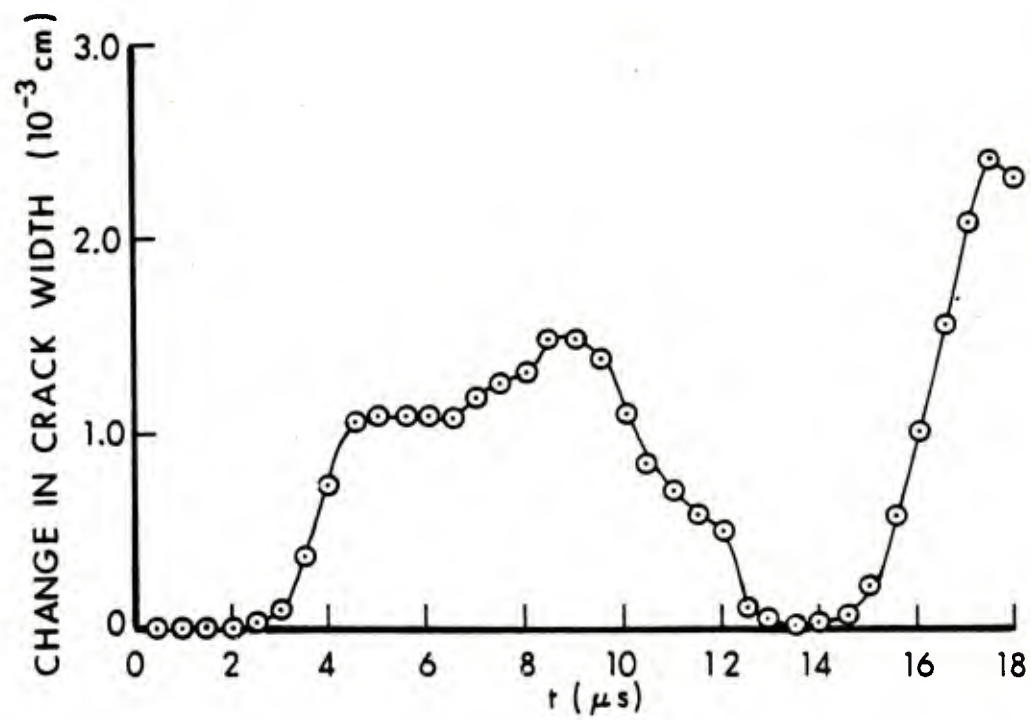


Figure 10. Crack Width as a Function of Time

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